

Why Occam's Razor

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Ensemble theories have received a lot of interest recently as a means of explaining a lot of the detailed complexity observed in reality by a vastly simpler description “every possibility exists” and a selection principle (*Anthropic Principle*) “we only observe that which is consistent with our existence”. In this paper I show why, in an ensemble theory of the universe, we should be inhabiting one of the elements of that ensemble with least information content that satisfies the anthropic principle. This explains the effectiveness of aesthetic principles such as Occam's razor in predicting usefulness of scientific theories. I also show, with a couple of reasonable assumptions about the phenomenon of consciousness, the linear structure of quantum mechanics can be derived.

Key words: Occam's razor, anthropic principle, ensemble theories, multiverse, failure of induction, foundation of quantum mechanics

1 INTRODUCTION

Wigner⁽¹⁾ once remarked on “the unreasonable effectiveness of mathematics”, encapsulating in one phrase the mystery of why the scientific enterprise is so successful. There is an aesthetic principle at large, whereby scientific theories are chosen according to their beauty, or simplicity. These then must be tested by experiment — the surprising thing is that the aesthetic quality of a theory is often a good predictor of that theory's explanatory and predictive power. This situation is summed up by William of Ockham, “Entities should not be multiplied unnecessarily”, known as Occam's Razor.

We start our search into an explanation of this mystery with the *anthropic principle*⁽²⁾. This is normally cast into either a weak form (that physical reality must be consistent with our existence as conscious, self-aware entities) or a

strong form (that physical reality is the way it is *because* of our existence as conscious, self-aware entities). The anthropic principle is remarkable in that it generates significant constraints on the form of the universe^(2, 3). The two main explanations for this are the *Divine Creator explanation* (the universe was created deliberately by God to have properties sufficient to support intelligent life), or the *Ensemble explanation*⁽³⁾ (that there is a set, or ensemble, of different universes, differing in details such as physical parameters, constants and even laws, however, we are only aware of such universes that are consistent with our existence). In the Ensemble explanation, the strong and weak formulations of the anthropic principle are equivalent.

Tegmark introduces an ensemble theory based on the idea that every self-consistent mathematical structure be accorded the ontological status of *physical existence*. He then goes on to categorize mathematical structures that have been discovered thus far (by humans), and argues that this set should be largely universal, in that all self-aware entities should be able to uncover at least the most basic of these mathematical structures, and that it is unlikely we have overlooked any equally basic mathematical structures.

An alternative ensemble approach is that of Schmidhuber's⁽⁴⁾ — the “Great Programmer”. This states that all possible halting programs of a universal Turing machine have physical existence. Some of these programs' outputs will contain self-aware substructures — these are the programs deemed interesting by the anthropic principle. Note that there is no need for the UTM to actually exist, nor is there any need to specify which UTM is to be used — a program that is meaningful on UTM₁ can be executed on UTM₂ by prepending it with another program that describes UTM₁ in terms of UTM₂'s instructions, then executing the individual program. Since the set of halting programs (finite length bitstrings) is isomorphic to the set of whole numbers \mathbb{N} , an enumeration of \mathbb{N} is sufficient to generate the ensemble that contains our universe. In a later paper⁽⁵⁾, Schmidhuber extends his ensemble to non-halting programs, and consider the consequences of assuming that this ensemble is generated by a machine with bounded resources.

Each self-consistent mathematical structure (member of the Tegmark ensemble) is completely described by a finite set of symbols, and a countable set of axioms encoded in those symbols, and a set of rules (logic) describing how one mathematical statement may be converted into another.¹ These axioms may be encoded as a bitstring, and the rules encoded as a program of a UTM that enumerates all possible theorems derived from the axioms, so each member of the Tegmark ensemble may be mapped onto a Schmidhuber one.² The

¹Strictly speaking, these systems are called recursively enumerable formal systems, and are only a subset of the totality of mathematics, however this seem in keeping with the spirit of Tegmark's suggestion

²In the case of an infinite number of axioms, the theorems must be enumerated using a dovetailer algorithm. The dovetailer algorithm is a means of walking an infinite level tree, such that each level is visited in finite time. An example is that for a n -ary tree, the nodes

Tegmark ensemble must be contained within the Schmidhuber one.

An alternative connection between the two ensembles is that the Schmidhuber ensemble is a self-consistent mathematical structure, and is therefore an element of the Tegmark one. However, all this implies is that one element of the ensemble may in fact generate the complete ensemble again, a point made by Schmidhuber in that the “Great Programmer” exists many times, over and over in a recursive manner within his ensemble. This is now clearly true also of the Tegmark ensemble.

2 UNIVERSAL PRIOR

In this paper, I adopt a Schmidhuber ensemble consisting of all infinite length bitstrings, denoted $\{0, 1\}^\infty$. I call these infinite length strings *descriptions*. By contrast to Schmidhuber, I assume a uniform measure over these descriptions — no particular string is more likely than any other. It can be shown that the cardinality of $\{0, 1\}^\infty$ is the same as the cardinality of the reals, c . This set cannot be enumerated by a dovetailer algorithm, rather the dovetailer algorithm enumerates all finite length prefixes of these descriptions. Whereas in Schmidhuber’s 1997⁽⁴⁾ paper, the existence of the dovetailer algorithm explains the ease with which the “Great Programmer” can generate the ensemble of universes, I merely assume the pre-existence of all possible descriptions. The information content of this complete set is precisely zero, as no bits are specified. It is ontologically equivalent to Nothing. This has been called the “zero information principle”.

Since some of these descriptions describe self aware substructures, we can ask the question of what these observers observe. An observer attaches sequences of *meanings* to sequences of prefixes of one of these strings. A meaning belongs to a countable set, which may be enumerated by the whole numbers. Thus the act of observation may be formalised as a map $O : [0, 1]^\infty \rightarrow \mathbb{N}$. If $O(x)$ is a *computable* (also known as a *recursive*) function, then $O(x)$ is equivalent to a Turing machine, for which every input halts. It is important to note that observers must be able to evaluate $O(x)$ within a finite amount of subjective time, or the observer simply ceases to be. The restriction to computable $O(x)$ connects this viewpoint with the original viewpoint of Schmidhuber.

Another interpretation of this scenario is a state machine, possibly finite, consuming bits of an infinite length string. As each bit is consumed, the current state of the machine is the meaning attached to the prefix read so far.

Under the mapping $O(x)$, some descriptions encode for identical meanings as other descriptions, so one should equivalence class the descriptions. In particular, strings where the bits after some bit number n are “don’t care” bits, are in fact equivalence classes of all strings that share the first n bits in common.

on the i th level are visited between steps n^i and $n^{i+1} - 1$.

One can see that the size of the equivalence class drops off exponentially with the amount of information encoded by the string. Under $O(x)$, the amount of information is not necessarily equal to the length of the string, as some of the bits may be redundant. The sum

$$P_O(s) = \sum_{p:O(p)=s} 2^{-|p|}, \quad (1)$$

where $|p|$ means the number of bits of p consumed by O in returning s , gives the size of the equivalence class of all descriptions having meaning s . This measure distribution is known as a *universal prior*, or alternatively a Solomonoff-Levin distribution, in the case where $O(x)$ is a universal prefix Turing machine⁽⁶⁾.

The quantity

$$\mathcal{C}_O(x) = -\log_2 P_O(O(x)) \quad (2)$$

is a measure of the information content, or *complexity* of a description x . If only the first n bits of the string are significant, with no redundancy, then it is easy to see $\mathcal{C}_O(x) = n$. Moreover, if O is a universal prefix Turing machine, then the coding theorem⁽⁶⁾ assures that $\mathcal{C}(x) \approx K(x)$, where $K(x)$ is the usual Kolmogorov complexity, up to a constant independent of the length of x .

If we assume the *self-sampling assumption*^(7, 8), essentially that we expect to find ourselves in one of the universes with greatest measure, subject to the constraints of the anthropic principle. This implies we should find ourselves in one of the simplest (in terms of \mathcal{C}_O) possible universes capable of supporting self-aware substructures (SASes). This is the origin of physical law — why we live in a mathematical, as opposed to a magical universe. This is why aesthetic principles, and Occam’s razor in particular are so successful at predicting good scientific theories. This might also be called the “minimum information principle”.

A final comment to highlight the distinction between this approach and Schmidhuber’s. Schmidhuber assumes that there is a given universal Turing machine U which generates the ensemble we find ourselves in. He even uses the term “Great Programmer” to underscore this. Ontologically, this is no more difficult than assuming there is an ultimate theory of everything — ie a final set of equations from which all of physics can be derived. Occam’s razor is a consequence of the resource constraints of U . In my approach, there is no given laws or global interpreter. By considering just the resource constraints of the observer, even in the case of the ensemble having a uniform measure, Occam’s razor still applies.

3 THE WHITE RABBIT PARADOX

An important criticism leveled at ensemble theories is what John Leslie calls *the failure of induction*^(9, §4.69). If all possible universes exist, then what is to

say that our orderly, well-behaved universe won't suddenly start to behave in a disordered fashion, such that most inductive predictions would fail in them. This problem has also been called the White Rabbit paradox⁽¹⁰⁾, presumably in a literary reference to Lewis Carrol.

This sort of issue is addressed by consideration of measure. We should not worry about the universe running off the rails, provided it is extremely unlikely to do so. Note that Leslie uses the term *range* to mean what we mean by *measure*. At first consideration, it would appear that there are vastly more ways for a universe to act strangely, than for it to stay on the straight and narrow, hence the paradox.

Evolution has taught us to be efficient classifiers of patterns, and to be robust in the presence of errors. It is important to know the difference between a lion and a lion-shaped rock, and to establish that difference in real time. Only a finite number of the description's bits are processed by the classifier, the remaining being "don't care" bits. Around each compact description is a cloud of completely random descriptions considered equivalent by the observer. The size of this cloud decreases exponentially with the complexity of the description.

This requirement imposes a significant condition on $O(x)$. Formally, each connected component of the preimage $O^{-1}(s)$ must be dense, ie have nonzero measure, in the space of descriptions.

Turing machines in general do not have this property of robustness against errors. Single bit errors in the input typically lead to wildly different outcomes. However, an artificial neural network, which is a computational model inspired by the brain does exhibit this robustness — leading to applications such as classifying images in the presence of noisy or extraneous data.

So what are the chances of the laws of physics breaking down, and of us finding ourselves in one of Lewis Carrol's creations? Such a universe will have a very complex description — for instance the coalescing of air molecules to form a fire breathing dragon would involve the complete specification of the states of some 10^{30} molecules, an absolutely stupendous amount of information, compared with the simple specification of the big bang and the laws of physics that gave rise to life as we know it. The chance of this happening is equally remote, via Eq. (1).

4 QUANTUM MECHANICS

In the previous sections, I demonstrate that formal mathematical systems are the most compressible, and have highest measure amongst all members of the Schmidhuber ensemble. In this work, I explicitly assume the validity of the *Anthropic Principle*, namely that we live in a description that is compatible with our own existence. This is by no means a trivial assumption — it is entirely possible that we are inhabiting a virtual reality where the laws of the observed world needn't be compatible with our existence. However, to date, the

Anthropic Principle has been found to be valid⁽²⁾.

In order to derive consequences of the Anthropic Principle, one needs to have a model of consciousness, or at very least some necessary properties that conscious observer must exhibit. I will explore the consequences of just two such properties of consciousness.

The first assumption to be made is that observers will find themselves embedded in a temporal dimension. A Turing machine requires time to separate the sequence of states it occupies as it performs a computation. Universal Turing machines are models of how humans compute things, so it is possible that all conscious observers are capable of universal computation. Yet for our present purposes, it is not necessary to assume observers are capable of universal computation, merely that observers are embedded in time.

The second assumption, which is related to Marchal's *computational indeterminism*⁽¹¹⁾, is that the simple mathematical description selected from the Schmidhuber ensemble describes the evolution of an ensemble of possible experiences. The actual world experienced by the observer is selected randomly from this ensemble. More accurately, for each possible experience, an observer exists to observe that possibility. Since it is impossible to distinguish between these observers, the internal experience of that observer is as though it is chosen randomly from the ensemble of possibilities. This I call the *Projection Postulate*.

The reason for this assumption is that it allows for very complex experiences to be generated from a very simple process. It is a very generalised form of Darwinian evolution, which exhibits extreme simplicity over *ex nihilo* creation explanations of life on Earth. Whilst by no means certain, it does seem that a minimum level of complexity of the experienced world is needed to support conscious experience of that world according to the anthropic principle.

This ensemble of possibilities at time t we can denote $\psi(t)$. Ludwig^(12,D1.1) introduces a rather similar concept of ensemble, which he equivalently calls *state* to make contact with conventional terminology. At this point, nothing has been said of the mathematical properties of ψ . I shall now endeavour to show that ψ is indeed an element from complex Hilbert space, a fact normally assumed as an axiom in conventional treatments of Quantum Mechanics.

The projection postulate can be modeled by a partitioning map $A : \psi \longrightarrow \{\psi_a, \mu_a\}$, where a indexes the allowable range of potential observable values corresponding to A , ψ_a is the subensemble satisfying outcome a and μ_a is the measure associated with ψ_a ($\sum_a \mu_a = 1$).

Finally, we assume that the generally accepted axioms of set theory and probability theory hold. Whilst the properties of sets are well known, and needn't be repeated here, the Kolmogorov probability axioms are⁽⁶⁾:

(A1) If A and B are events, then so is the *intersection* $A \cap B$, the *union* $A \cup B$ and the *difference* $A - B$.

(A2) The *sample space* S is an event, called the *certain event*, and the *empty*

set \emptyset is an event, called the *impossible* event.

(A3) To each event E , $P(E) \in [0, 1]$ denotes the *probability* of that event.

(A4) $P(S) = 1$.

(A5) If $A \cap B = \emptyset$, then $P(A \cup B) = P(A) + P(B)$.

(A6) For a decreasing sequence $A_1 \supset A_2 \supset \dots \supset A_n \dots$ of events with $\bigcap_n A_n = \emptyset$, we have $\lim_{n \rightarrow \infty} P(A_n) = 0$.

Consider now the projection operator $\mathcal{P}_{\{a\}} : V \longrightarrow V$, acting on an ensemble $\psi \in V$, V being the set of all such ensembles, to produce $\psi_a = \mathcal{P}_{\{a\}}\psi$, where $a \in S$ is an outcome of an observation. We have not at this stage assumed that $\mathcal{P}_{\{a\}}$ is linear. Define addition for two distinct outcomes a and b as follows:

$$\mathcal{P}_{\{a\}} + \mathcal{P}_{\{b\}} = \mathcal{P}_{\{a,b\}}, \quad (3)$$

from which it follows that

$$\mathcal{P}_{A \subset S} = \sum_{a \in A} \mathcal{P}_{\{a\}} \quad (4)$$

$$\mathcal{P}_{A \cup B} = \mathcal{P}_A + \mathcal{P}_B - \mathcal{P}_{A \cap B} \quad (5)$$

$$\mathcal{P}_{A \cap B} = \mathcal{P}_A \mathcal{P}_B = \mathcal{P}_B \mathcal{P}_A. \quad (6)$$

These results extend to continuous sets by replacing the discrete sums by integration over the sets with uniform measure. Here, as elsewhere, we use Σ to denote sum or integral respectively as the index variable a is discrete or continuous.

Let the ensemble $\psi \in V \equiv \{\mathcal{P}_A \psi | A \subset S\}$ be a “reference state”, corresponding to the certain event. It encodes information about the whole ensemble. Denote the probability of a set of outcomes $A \subset S$ by $P_\psi(\mathcal{P}_A \psi)$. Clearly

$$P_\psi(\mathcal{P}_S \psi) = P_\psi(\psi) = 1 \quad (7)$$

by virtue of (A4). Also, by virtue of Eq. (5) and (A4),

$$P_\psi((\mathcal{P}_A + \mathcal{P}_B)\psi) = P_\psi(\mathcal{P}_A \psi) + P_\psi(\mathcal{P}_B \psi) \quad \text{if } A \cap B = \emptyset. \quad (8)$$

Assume that Eq. (8) also holds for $A \cap B \neq \emptyset$ and consider the possibility that A and B can be identical. Eq. (8) may be written:

$$P_\psi((a\mathcal{P}_A + b\mathcal{P}_B)\psi) = aP_\psi(\mathcal{P}_A \psi) + bP_\psi(\mathcal{P}_B \psi), \forall a, b \in \mathbb{N}. \quad (9)$$

Thus, the set V naturally extends by means of the addition operator defined by Eq. (3) to include all linear combinations of observed states, at minimum over the natural numbers. If $A \cap B \neq \emptyset$, then $P_\psi((\mathcal{P}_A + \mathcal{P}_B)\psi)$ may exceed unity, so

clearly $(\mathcal{P}_A + \mathcal{P}_B)\psi$ is not necessarily a possible observed outcome. How should we interpret these new “nonphysical” states?

At each moment that an observation is possible, an observer faces a choice about what observation to make. In the Multiverse, the observer differentiates into multiple distinct observers, each with its own measurement basis. In this view, there is no *preferred basis*⁽¹³⁾.

The expression $P_\psi((a\mathcal{P}_A + b\mathcal{P}_B)\psi)$ must be the measure associated with a observers choosing to partition the ensemble into $\{A, \bar{A}\}$ and observing an outcome in A and b observers choosing to partition the ensemble into $\{B, \bar{B}\}$ and seeing outcome B . The coefficients a and b must be drawn from a measure distribution over the possible choices of measurement. The most general measure distributions are complex, therefore the coefficients, in general are complex⁽¹⁴⁾. We can comprehend easily what a positive measure means, but what about complex measures? What does it mean to have an observer with measure -1 ? It turns out that these non-positive measures correspond to observers who chose to examine observables that do not commute with our current observable A . For example if A were the observation of an electron’s spin along the z axis, then the states $|+\rangle + |-\rangle$ and $|+\rangle - |-\rangle$ give identical outcomes as far as A is concerned. However, for another observer choosing to observe the spin along the x axis, the two states have opposite outcomes. This is the most general way of partitioning the Multiverse amongst observers, and we expect to observe the most general mathematical structures compatible with our existence.

The probability function P can be used to define an inner product as follows. Our reference state ψ can be expressed as a sum over the projected states $\psi = \sum_{a \in S} \mathcal{P}_{\{a\}}\psi \equiv \sum_{a \in S} \psi_a$. Let $V^* = \mathcal{L}(\psi_a)$ be the linear span of this basis set. Then, $\forall \phi, \xi \in V$, such that $\phi = \sum_{a \in S} \phi_a \psi_a$ and $\xi = \sum_{a \in S} \xi_a \psi_a$, the inner product $\langle \phi, \xi \rangle$ is defined by

$$\langle \phi, \xi \rangle = \sum_{a \in S} \phi_a^* \xi_a P_\psi(\psi_a). \quad (10)$$

It is straightforward to show that this definition has the usual properties of an inner product, and that ψ is normalized ($\langle \psi, \psi \rangle = 1$). The measures μ_a are given by

$$\begin{aligned} \mu_a = P_\psi(\psi_a) &= \langle \psi_a, \psi_a \rangle \\ &= \langle \psi, \mathcal{P}_a \psi \rangle \\ &= |\langle \psi, \hat{\psi}_a \rangle|^2, \end{aligned} \quad (11)$$

where $\hat{\psi}_a = \psi_a / \sqrt{P_\psi(\psi_a)}$ is normalised.

Until now, we haven’t used axiom (A6). Consider a sequence of sets of outcomes $A_0 \supset A_1 \dots$, and denote by $A \subset A_n \forall n$ the unique maximal subset (possibly empty), such that $\bar{A} \cap_n A_n = \emptyset$. Then the difference $\mathcal{P}_{A_i} - \mathcal{P}_A$ is well

defined, and so

$$\begin{aligned}
\langle (\mathcal{P}_{A_i} - \mathcal{P}_A)\psi, (\mathcal{P}_{A_i} - \mathcal{P}_A)\psi \rangle &= P_\psi((\mathcal{P}_{A_i} - \mathcal{P}_A)\psi) \\
&= P_\psi((\mathcal{P}_{A_i} + \mathcal{P}_{\bar{A}} - \mathcal{P}_S)\psi) \\
&= P_\psi(\mathcal{P}_{A_i \cap \bar{A}}).
\end{aligned} \tag{12}$$

By axiom (A6),

$$\lim_{n \rightarrow \infty} \langle (\mathcal{P}_{A_i} - \mathcal{P}_A)\psi, (\mathcal{P}_{A_i} - \mathcal{P}_A)\psi \rangle = 0, \tag{13}$$

so $\mathcal{P}_{A_i}\psi$ is a Cauchy sequence that converges to $\mathcal{P}_A\psi \in V$. Hence V is complete under the inner product (10). It follows that V^* is complete also, and is therefore a *Hilbert* space.

The most general form of evolution of ψ in continuous time is given by:

$$\frac{d\psi}{dt} = \mathcal{H}(\psi). \tag{14}$$

Some people may think that discreteness of the world's description (ie of the Schmidhuber bitstring) must imply a corresponding discreteness in the dimensions of the world. This is not true. Between any two points on a continuum, there are an infinite number of points that can be described by a finite string — the set of rational numbers being an obvious, but by no means exhaustive example. Continuous systems may be made to operate in a discrete way, electronic logic circuits being an obvious example. For the sake of connection with conventional quantum mechanics, we will assume that time is continuous. A discrete time formulation can also be derived, in which case we need a difference equation instead of Eq. (14). Other possibilities also exist, such as the rational numbers example mentioned before. The theory of *time scales*⁽¹⁵⁾ could provide a means of developing these other possibilities.

Axiom (A3) constrains the form of the evolution operator \mathcal{H} . Since we suppose that ψ_a is also a solution of Eq. 14 (ie that the act of observation does not change the physics of the system), \mathcal{H} must be linear. The certain event must have probability of 1 at all times, so

$$\begin{aligned}
0 &= \frac{dP_{\psi(t)}(\psi(t))}{dt} \\
&= d/dt \langle \psi, \psi \rangle \\
&= \langle \psi, \mathcal{H}\psi \rangle + \langle \mathcal{H}\psi, \psi \rangle \\
\mathcal{H}^\dagger &= -\mathcal{H},
\end{aligned} \tag{15}$$

i.e. \mathcal{H} is i times a Hermitian operator.

5 Discussion

A conventional treatment of quantum mechanics (see eg Shankar⁽¹⁶⁾) introduces a set of 4-5 postulates that appear mysterious. In this paper, I introduce a model of observation based on the idea of selecting actual observations from an ensemble of possible observations, and can derive the usual postulates of quantum mechanics aside from the *Correspondence Principle*.³ Even the property of *linearity* is needed to allow disjoint observations to take place simultaneously in the universe. Weinberg^(18, 19) experimented with a possible non-linear generalisation of quantum mechanics, however found great difficulty in producing a theory that satisfied causality. This is probably due to the nonlinear terms mixing up the partitioning $\{\psi_a, \mu_a\}$ over time. It is usually supposed that causality⁽³⁾, at least to a certain level of approximation, is a requirement for a self-aware substructure to exist. It is therefore interesting, that relatively mild assumptions about the nature of SASEs, as well as the usual interpretations of probability and measure theory lead to a linear theory with the properties we know of as quantum mechanics. Thus we have a reversal of the usual ontological status between Quantum Mechanics and the *Many Worlds Interpretation*⁽²⁰⁾.

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I would also like to thank the anonymous reviewer for suggesting Ludwig’s book⁽¹²⁾. Whilst the intuitive justification in that book is very different, there is a remarkable congruence in the set of axioms chosen to the ones presented in this paper.

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³The *Correspondence Principle* states that classical state variables are represented in the quantum formulation by replacing appropriately $x \rightarrow X$ and $p \rightarrow -i\hbar d/dx$. Stenger⁽¹⁷⁾ has developed a theory based on fundamental symmetries that explains the Correspondence Principle.

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